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It has been recently shown that meta-stable domain walls exist in high-density QCD ($\mu \neq 0$) as well as in QCD with large number of colors ($N_c \to \infty$), with the lifetime being exponentially long in both cases. Such metastable domain walls may exist in our world as well, especially in hot hadronic matter with temperature close to critical. In this paper we discuss what happens if a bubble made of such wall is created in heavy ion collisions, in the mixed phase between QGP and hadronic matter. We show it will further be expanded to larger volume $\sim 20 fm^3$ by the pion pressure, before it disappears, either by puncture or contraction. Both scenarios leave distinctive experimental signatures of such events, negatively affecting the interference correlations between the outgoing pions.

I. INTRODUCTION

Domain walls are common in field theory, the simplest in the family of topological objects. If they are configurations of fields interpolating between two distinct vacua, the domain wall obviously cannot decay. However even in theories with a *single* vacuum domain-wall configurations may nevertheless exist. Such domain walls are solitons which usually require a discrete-type of symmetry. In QCD such a symmetry indeed exists and corresponds to the discrete rotations of the so-called θ angle $\theta \to \theta + 2\pi n$.

The θ parameter is not a dynamical field in QCD*, and therefore it can not make a domain wall by itself. However, the θ angle appears in the low energy description of QCD together with the physical $\eta'(x)$ field in a very special combination $(\theta - i \log DetU)$ where the unitary matrix U describes the pseudo-goldstone fields and the singlet $\eta'(x)$ field. Therefore, the domain walls which may exist due to the discrete symmetry discussed above, can be realized by the physical $\eta'(x)$ field. If the η' field is the light field, all calculations are under theoretical control and one can argue that the life time of the domain wall is parametrically large in large N_c limit $\tau_{life} \sim \exp(N_c^2)$, [1], as well as in large μ limit $\tau_{life} \sim \exp(\mu^{2+b}), b = \frac{11}{3}N_c - \frac{2}{3}N_f$ [2] if one considers the high density QCD with large μ.

In our world with $N_c = 3$ the η' meson is not a light particle, although it may become lighter for sufficiently hot hadronic matter as a result of partial restoration of $U(1)_A$ symmetry around the QCD phase transition, as argued in [3,4]. At the moment, we do not know what exactly happens with this excitation at such conditions, there is no theoretical control or sufficient lattice data. However, as was argued in Ref. [1], the domain wall in the theory with $N_c = 3$ can still be *classically stable*. In reality (at quantum level), it is of course only *metastable*, but with a life time much longer than a typical QCD time scale, $\sim \Lambda_{QCD}^{-1}$. The argument of Ref. [1] is based on the observation that one should not naively compare $m_{\eta'}$ mass with a typical hadronic scale which is the same order of $1GeV$. Instead, one should compare the vacuum energy density due to the gluon degrees of freedom (it can be explicitly expressed in terms of the gluon condensate, $E = \langle \frac{b\alpha_s}{32\pi} G^2 \rangle \sim N_c^2$) with the corresponding contribution due to the η' excitation when the relevant dimensionless phase $\frac{\eta'}{f_{\eta'}}$ becomes order of one. Therefore, $\frac{1}{2}m_{\eta'}^2 \eta'^2 \sim \frac{1}{2}m_{\eta'}^2 f_{\eta'}^2 \sim \frac{\partial^2}{\partial\theta^2}E \sim N_c^2/N_c^2 \sim 1 \ll E$. Exactly this inequality prevents the domain walls from the classically allowed fast decay as discussed in [1]. In what follows we assume that η' domain wall is classically stable object, and therefore, it decays through the quantum tunneling process with exponentially large lifetime† which is longer than any other time scales existing in the heavy ion collisions.

The main point of this letter is the observation that if such domain walls indeed exist in QCD, they can be produced and studied in heavy ion collisions. At high collision energies (SPS, RHIC) the excited matter is assumed to be produced in the QGP phase, and then cools down, spending significant time ($\sim 5fm/c$) in the so called *mixed phase*. Small bubbles made of domain walls can be produced by thermal fluctuations at this stage: currently we are not able to provide any quantitative estimates of the probability of this to happen, as it would require an understanding of the "out of equilibrium" physics. However, one may not expect a huge suppression for the pro-

*We do not discuss axions in this paper.

†Recall familiar soap film bubbles: those are metastable as well, but they do exist for minutes with sizes of many cm, which is very impressive if expressed in terms microscopic (atomic) units.

duction of a $\sim fewfm$ size bubble due to the general arguments that all dimensional parameters of the problem have one and the same $\sim 1fm$ scale during this period. In fact the domain wall tension σ [1] :

$$\sigma \simeq \frac{\pi^2 f_\pi \sqrt{E}}{2\sqrt{2}N_c} \simeq (200MeV)^3 \simeq (1fm)^{-3}. \qquad (1)$$

is not large enough to upset energetics at $T = T_c \approx 160\ MeV$. Our main observation is that after being produced, a small ($1fm$ in size) bubble would *grow substantially in size*, driven by the pion pressure, spending a significant extra time $(\sim 5)fm/c$ as a relatively large macroscopic object, before releasing all its content and disappear.

How can these large bubbles be observed if produced? Our suggestion is to monitor (on event-by-event basis) the strength of the two-pion Bose-Einstein correlation function, $\lambda_*(p)$, see below, which it turns out to be very sensitive to any macroscopically large object with long enough life time. All pions which are eventually emitted from such an object will be completely incoherent with the rest of pions.

Closing the Introduction we should mention that related but different macroscopically large configurations were also discussed in Ref. [5] in the context of the decay of the metastable vacua possibly created in heavy ion collisions. The difference with this work is quite significant: we do not consider a metastable vacuum, but rather a metastable wall making a bubble.

II. BUBBLE DYNAMICS

Effective Lagrangian for bubble motion is

$$L = \frac{4\pi\sigma R^2(t)}{2}\dot{R}^2(t) - 4\pi\sigma R^2(t) + \frac{4\pi}{3}R^3(t)P_\pi \qquad (2)$$

where σ is the wall surface tension (1) and $R(t)$ is bubble radii. The last term describes the effective pressure induced by pions, which competes with the surface tension and try to expand the bubble. It appears because pions are scattered back, from the domain wall into the bubble. The probability of this to happen $P(k_r)$, the reflection coefficient of π meson with momentum k_r off a domain wall, enters the pion pressure

$$P_\pi = \int 2k_r v_r n_\pi(\vec{k})P(k_r)\frac{d^3k}{(2\pi)^3}, \qquad (3)$$

($P(k_r)$ will be estimated in the next section.) We assume the bubble is a spherically symmetric object such that we keep only the r component, v_r is the velocity of the pions; $n_\pi(\vec{k})$ is the pion density, to be discussed below. The equation of motion is then

$$\ddot{R}(t) = -\frac{\dot{R}^2}{R} - \frac{2}{R} + \frac{P_\pi}{\sigma}, \qquad (4)$$

which has perfect physical meaning as the last term describes the acceleration of the bubble's surface. Indeed, P_π can be interpreted as the force/area applied to the bubble, and σ can be interpreted as mass/area of the bubble.

The bubble radius $R(t)$ is one time-dependent variable: the second one is the temperature of the hadronic gas inside the bubble $T(t)$. (We assume that pions rescattered back from the bubble wall are quickly equilibrated with the rest of pions inside it.) We therefore need the second equation, which we get simply from the energy conservation. One can identify the following terms in the energy change of the bubble

$$\dot{E}_{gas} = -\dot{E}_{leakage} - \dot{E}_{wall} \qquad (5)$$

where

$$\dot{E}_{gas} = \frac{d}{dt}(c_{SB}T^4\frac{4\pi R^3(t)}{3}) \qquad (6)$$

and $c_{SB} = \pi^2/10$ for massless pion Bose gas‡.

$$\dot{E}_{leakage} = 4\pi R^2 \int k_r v_r n_\pi(\vec{k})(1-P(k_r))\frac{d^3k}{2(\pi)^3} \qquad (7)$$

$$\dot{E}_{wall} = 8\pi\sigma R\dot{R} \qquad (8)$$

After the replacement $\langle v_r k_r\rangle \to \frac{1}{3}\langle k\rangle$ in eq.(7) (brackets imply the averaging over ensemble with temperature T), the corresponding equation finally gets the form

$$\frac{4\dot{T}}{3T} + \frac{\dot{R}}{R} = -\frac{1}{3R} + \frac{P_\pi}{2c_{SB}T^4R} - \frac{2\sigma\dot{R}}{R^2c_{SB}T^4} \qquad (9)$$

If the rhs is set to zero, the bubble evolution gives just $T^4R^3 = const$, as energy conservation would require. The negative sign of the rhs tends to compress the bubble.

Our main observation is that after being produced, small bubble would be forced to grow substantially, provided some conditions (to be specified below) are met. This expansion may happen due to two different reasons.

The first, if a bubble happen to contain a significant portion of its volume by QGP, it will expand simply because while it is transformed into hadronic matter (approximated by a pion gas) it occupies much larger volume. The simplest (but naive) estimate of such expansion ratio R_1^{exp} is obtained if both the QGP and the pion gas are treated as ideal massless gases; then it is just the ratio of degrees of freedom (DOF) of QGP ($c_{QGP} = 47.5, N_c = N_f = 3$) to that of the pion gas ($c_\pi = 3$). If so, the volume is expected to grow by an

‡It is $9/\pi^2$ in Boltzmann approximation we will use for simplicity.

order of magnitude. In reality, we have the resonance gas, not the pion one, with more degrees of freedom, and partons are not really massless. So R_1^{exp} is not $c_{QGP}/c_\pi \simeq 16$, but somewhat smaller.

The second mechanism driving expansion works in hadronic phase. We will show in the next section that the thermal pions are rather effectively reflected by the domain wall, so that they are effectively trapped inside the bubble. It leads to another expansion ratio $R_2^{exp} = s(T_c)/s(T_{min})$ where the entropy densities at two temperatures are included. The latter one, T_{min}, should correspond to final mechanically equilibrium bubble satisfying the following condition

$$p(T_min) = \sigma/R_f \tag{10}$$

The larger the final bubble radius R_f, the lower this pressure. The lifetime of this equilibrium is determined either by partial leakage of pions, or by the bubble lifetime, whatever is shorter.

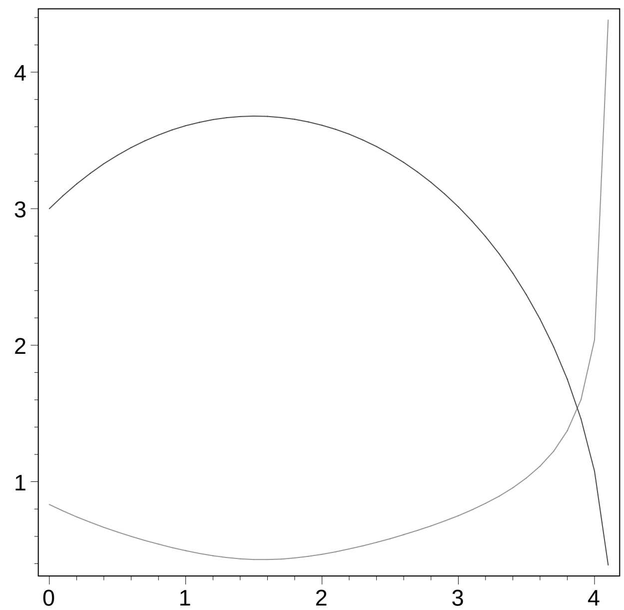

FIG. 1. An example of the solution of equations of motion of the bubble dynamics, described in the text. The solid line is the dependence of the bubble radius $R(t)$, in fm, versus time t, in fm. We assume $R(t=0) = 3fm$ which corresponds to the initially small bubble about $1.2fm$ expanded due to the first mechanism, $1.2fm \cdot (\frac{c_{QGP}}{c_\pi})^{1/3} \simeq 3fm$ up to the relatively large size $R(t=0) \simeq 3fm$. The dotted line represents the corresponding evolution of the internal temperature T (in fm^{-1}).

Equations (4) and (9) derived above, describe the dynamics of bubbles in this hadronic phase and correspond to the second stage of expansion determined by R_2^{exp}. We solved equations (4) and (9) numerically to demonstrate the effect of this expansion, see Fig.1. Let us only comment on T evolution. It starts at the critical value $T_c = 165\, MeV = .83\ fm^{-1}$ and then cools down as the bubble expands. However, the equations indicate a secondary heating as the bubble walls collapses. We will not dwell on it, but note that quite similar phenomena are known elsewhere in physics, e.g. in the so called sonoluminescence: so it may be not just an artifact of the equation solution.

III. PION SCATTERING ON A DOMAIN WALL

In the discussions presented above we introduced in our formula (3) one essential parameter, the reflection probability $P(k_z)$ which is, by definition, the probability of reflection of π meson off a domain wall. The behavior of this parameter has not been specified yet, we shall estimate $P(k_z)$ now.

First of all, we start from the low energy effective Lagrangian when π and η' fields are described by the unitary matrix in the simplified version of the theory when $N_f = 2, m_u = m_d$:

$$U = \exp\left(i\frac{\sqrt{2}\pi^a\sigma^a}{f_\pi} + i\frac{\sqrt{2}\eta'}{f_{\eta'}}\right),\ UU^\dagger = 1,\ , \tag{11}$$

where σ^a are the Pauli matrices, π^a is the triplet, and $f_\pi \simeq f_{\eta'} \simeq 133MeV$. In terms of U the low energy effective Lagrangian is given by [6]:

$$L = \frac{f_\pi^2}{8}Tr(\partial_\mu U^\dagger \partial_\mu U) + \frac{1}{2}MTr(U + U^\dagger) + E\cos(\frac{i\log DetU - \theta}{N_c}), \tag{12}$$

where all dimensional parameters in this Lagrangian are expressed in terms of QCD vacuum condensates, and are well-known: $M = m_q|\langle\bar{\psi}\psi\rangle|$; the constant E is related to the gluon condensate $E = \langle\frac{b\alpha_s}{32\pi}G^2\rangle$. The first two terms describe the standard expression for the effective chiral Lagrangian; the last term describes the η' field. This term is not uniquely fixed by the symmetry; however the only important element for the following discussions is the manifest 2π periodicity for the η' term as well as an appearance of the scale E which remains finite in the chiral limit. A specific cos form for this term is not essential; however it will be used for all numerical estimates presented below.

As we discussed in the Introduction the theory (12) supports the metastable domain walls. We refer the reader to the original paper [1] for the discussions of the specific properties of the domain wall; the only what we need to know here about the QCD domain walls is their following general properties:

a). The QCD domain wall is described by two dimensionless phases, $\phi_S(z)$ describing the isotopical "singlet", and $\phi_T(z)$ isotopical "triplet" fields. These fields correspond to the dynamical η' (singlet) and pion π^0 (triplet) fields defined in (11); they depend only on one variable

z. Both dimensionless fields $\phi_S(z), \phi_T(z)$ interpolate between zero and 2π when z varies from $-\infty$ to $+\infty$ and can be expressed in terms of the original dimensional fields η', π^0 as follows.

$$\phi_{S,T} = \phi_u \pm \phi_d, \ \eta' = \frac{f_\pi}{2\sqrt{2}}\phi_S(z), \ \pi^0 = \frac{f_\pi}{2\sqrt{2}}\phi_T(z). \quad (13)$$

b). Metastable domain walls of a minimal energy correspond to the transitions from $(\phi_u, \phi_d)|_{z=-\infty} = (0,0)$ to $(\phi_u, \phi_d)|_{z=+\infty} = (2\pi, 0)$. In the limit $m_u = m_d$ the transition to $(\phi_u, \phi_d) = (0, 2\pi)$ have the same energy and there is a degeneracy. In reality $m_d > m_u$, and the transition to $(\phi_u, \phi_d) = (2\pi, 0)$ is the only stable transition.
c).Domain wall has a sandwich-like structure: ϕ_S substantially varies on the scale $z \sim m_{\eta'}^{-1}$ while ϕ_T varies on considerably larger scale $z \sim m_\pi^{-1} \gg m_{\eta'}^{-1}$ where ϕ_S is already close to its vacuum values $0, 2\pi$. Therefore, one can say that the η' transition is sandwiched in the pion transition. In spite of the fact that ϕ_T is much wider than ϕ_S the main contribution to the wall tension (1) comes from the ϕ_S transition. Anti-soliton corresponds to the transition from $(\phi_u, \phi_d)|_{z=-\infty} = (0,0)$ to $(\phi_u, \phi_d)|_{z=+\infty} = (-2\pi, 0)$.

Our next step is to consider small oscillations in the pion and η' fields about the static QCD domain wall solution(13):

$$\vec{\pi}(x_\mu) \to \left(\pi_1(x_\mu), \pi_2(x_\mu), \frac{f_\pi}{2\sqrt{2}}\phi_T(z) + \pi_3(x_\mu)\right);$$
$$\eta'(x_\mu) \to \frac{f_\pi}{2\sqrt{2}}\phi_S(z) + \eta'(x_\mu), \quad (14)$$

where $\phi_T(z)$ and $\phi_S(z)$ are solutions known numerically [1] and qualitatively described above. It is clear that a particle scattering by a planar domain wall reduces to a one-dimensional scattering problem. To further simplify things we neglect scattering of η' particles which can not play an important role due to its larger mass.

First, let us consider the simple case of scattering of the neutral π_0 meson off the domain wall. With the plane wave ansatz, $\pi_3(x_\mu) = \pi_3(z)\exp(-i\omega t + ik_x x + ik_y y)$, the field equation for $\pi_3(z)$ which follows from the chiral Lagrangian (12), can be written as

$$\left(\frac{d^2}{dz^2} + k_z^2 - U_{\pi^0}(z)\right)\pi_3(z) = 0,$$
$$k_z^2 \equiv \omega^2 - k_x^2 - k_y^2 - m_\pi^2, \quad (15)$$

where the effective potential $U_{\pi^0}(z)$ for the problem is expressed in terms of the domain wall profile functions $\phi_T(z)$ and $\phi_S(z)$:

$$U_{\pi^0}(z) = -m_\pi^2\left(1 - \cos\frac{\phi_T(z)}{2}\cos\frac{\phi_S(z)}{2}\right) \quad (16)$$

Equations (15), (16) can not be solved analytically, however, a qualitative behavior of the reflection coefficient $P(k_z)$ can be easily understood from the following arguments. If the particle wavelength is much smaller than the thickness of the wall, $k_z \gg m_\pi$, the reflection coefficient is exponentially small according to the standard semiclassical arguments. In the opposite, the long-wavelength limit $k_z \to 0$, the wall potential (16) can be adequately approximated by a $\delta(z)$ function,

$$U_{\pi^0}(z) \simeq -\alpha\delta(z), \quad \alpha = -\int_{-\infty}^{+\infty} U_{\pi^0}(z)dz \sim m_\pi. \quad (17)$$

The scattering problem with a $\delta(z)$ function potential is easily solved; the reflection coefficient is

$$P(k_z) = \frac{\alpha^2}{\alpha^2 + (2k_z)^2}. \quad (18)$$

As expected, the reflection coefficient does not depend on the sign of the potential $U_{\pi^0}(z)$, and $P(k_z) \to 1$ for small $k_z \to 0$.

Our next task is an analysis of a similar problem for the charged π^+, π^- components of $\vec{\pi}$ field. Some technical complications arise here due to the fact that the kinetic term $\sim Tr(\partial_\mu U^\dagger \partial_\mu U)$ in the low energy Lagrangian (12) is not reduced to the canonical kinetic terms $\sim 1/2(\partial_\mu\vec{\pi})^2$ for the individual components when the expansion in (12) is made about a nontrivial classical configuration. Instead, the kinetic term is given by

$$\frac{1}{2}Tr(\partial_\mu U^\dagger \partial_\mu U) = (\partial_\mu\theta)^2 + (\partial_\mu\phi)^2 + \sin^2\theta(\partial_\mu\vec{n})^2, \quad (19)$$

where, in order to simplify formula (19), we introduce new variables $\theta(x), \phi(x), \vec{n}(x)$ expressed in terms of the original fields $\vec{\pi}, \eta'$ (11) as follows:

$$|\vec{\pi}| \equiv \sqrt{\vec{\pi}^2}; \quad \vec{n} \equiv \frac{\vec{\pi}}{|\vec{\pi}|}; \quad \theta \equiv \frac{\sqrt{2}|\vec{\pi}|}{f_\pi}; \quad \phi \equiv \frac{\sqrt{2}\eta'}{f_\pi}. \quad (20)$$

After some algebra we arrive to the following expression (analogous to (15,16)) describing the scattering of charged π_1, π_2 components off the wall.

$$\left(\frac{d^2}{dz^2} + k_z^2 - U_{\pi^\pm}(z)\right)\pi_i(z) = 0, \quad i = 1, 2$$
$$k_z^2 \equiv \omega^2 - k_x^2 - k_y^2 - m_\pi^2, \quad (21)$$

where the effective potential $U_{\pi^\pm}(z)$ for the charged $\pi_{1,2}$ components is expressed in terms of the same profile functions $\phi_T(z)$ and $\phi_S(z)$ as follows

$$U_{\pi^\pm}(z) = m_\pi^2\left(\frac{\phi_T(z)}{2}\frac{\cos\frac{\phi_S(z)}{2}}{\sin\frac{\phi_T(z)}{2}} - 1\right) + \delta U_{\pi^\pm}(z), \quad (22)$$

where $\delta U_{\pi^\pm}(z)$ is a quite complicated operator defined in the Appendix and numerically will be ignored for the qualitative discussions which follow. Note, that in all formulae presented above the profile functions $\phi_S(z), \phi_T(z)$ describing the wall are defined in the region $-\infty < z < 0$

when $0 < \phi_S(z), \phi_T(z) < \pi$. For the positive z, by symmetry, one should replace $\phi_{S,T} \to 2\pi - \phi_{S,T}$ as explained in [1]. Qualitative properties of the potential $U_{\pi^\pm}(z)$ are the same as $U_{\pi^0}(z)$ discussed earlier (16), namely, a long-wavelength particle is reflected from the wall with a very high probability, while in the short-wavelength limit the reflection probability is nearly zero. The reflection coefficient is slightly different for the charged and neutral pions; however, in what follows we neglect that difference and approximate the reflection coefficient as follows

$$P(k_z) \simeq e^{-\frac{k_z}{k_0}}, \quad k_0 \simeq m_\pi. \tag{23}$$

This is our final expression which was used in the previous section, see eqs.(3), (7), for the numerical estimates presented on Fig1.

IV. EXPERIMENTAL OBSERVABLE

Our main idea of the experimental observation of bubbles is to make use of the intensity interferometry of pions, due to their Bose-Einstein statistics. This method, also known as HBT (Hanbury-Brown-Twiss) interferometry, has been originally introduced for measuring the angular diameters of stars. For pions it has been first observed in $\bar{p}p$ annihilation, and explained in the famous paper by Goldhaber et al [7]. Since early 70's it has been used to extract source sizes of hadronic fireballs, see e.g. one of the early papers on the subject by one of us [8]. Recently it has been argued [9] that one can use pion interferometry as a sensitive tool to detect possible increase of the η' production in heavy ion collisions. We now extended the same reasoning for the observation of the bubble production.

If a bubble is produced, it exists for some lifetime and then decay. It can either happen due to (i) puncture of the wall, or (ii) simple contraction, as discussed above. Both scenarios lead to similar observable signature.

In the case of puncture, the bubble walls decays into its underlying fields, the η'. Since it happens after a freeze-out time for most pions not belonging to the bubble, most of these η' would not be re-absorbed, and decay normally, most often into 5 pions. One may argue (direct following ref. [9]) that there exist an important observable distinction between those pions and the rest of them produced from the fireball. Large lifetime of η', η make their products *incoherent* to other pions, reducing the HBT peak in two-pion spectra. It happens, because the inverse of the η' lifetime is much smaller than experimental energy resolution of the detectors.

The second (and, as our estimates suggest, see Section II, more probable) mechanism for the bubble decay is described above: it is due to a eventual collapse of the bubble due to pion leakage. In this case bubble surface contracts and large population of η' is not expected. However this processes takes long enough time, $\sim$ 5 fm/c, so the bubble itself plays the role of long-lived object. Again, the inverse of this time is likely to be below the detector resolution. As a result, most of the pions trapped in these quasi-stationary bubbles and released later become incoherent with others pions. For this reason the pions from bubbles lead to the same effect of not producing a HBT peak in two-pion spectra.

The strength of the HBT correlation is characterized by the effective intercept parameter $\lambda_*(p)$ defined as follows [9]

$$\lambda_*(p) = \left[\frac{N_{direct}(p)}{N_{direct}(p) + N_{delayed}(p)}\right]^2, \tag{24}$$

where $N_{direct}(N_{delayed}(p))$ is the one- particle- invariant momentum distribution of the "core" direct ("halo" or delayed) pions. As was discussed in [9], a substantial increase in the η' production will result in appearance of a hole *in the low p_t region* of the effective intercept parameter $\lambda_*(p)$ centered around $p_t \simeq 138MeV$ which represents the average p_t of the pions coming from η' decay. If a bubble is punctured, we also expect the same signature, due to an increase number of η' to be produced with low p_t.

However, if the second mechanism of decay (slow inflation and deflation of the bubble) is prevailed, the effect of decreasing $\lambda_*(p)$ is different. This is because the emission of pions from the bubble happen through the walls, and low energy pions are expected to be nearly completely trapped (note, the reflection coefficient is close to one for the cold pions and zero for the hot ones, see previous Section III). So we expect to see a decreasing $\lambda_*(p)$ at larger p instead.

To make a numerical estimate of the effect, we note that the parameter λ_* equals to 1 for completely coherent pions and reduced to about 0.5 in usual experimental conditions[§]. If bubbles are produced, the intercept λ_* would be additionally reduced by the factor $(1 - f_{bubble})^2$ as follows from (24). Here f_{bubble} is the fraction of pions coming from the bubbles. this parameter can be easily estimated as follows. The bubble energy is order of $E_{bubble} \simeq 4\pi R^2 \sigma \sim 60GeV$, where we use $R \sim 5fm$ and $\sigma \sim 1fm^{-3}$ (1). If all the energy accumulated in the wall of the bubble will go to the production of the η' mesons (which will result in additional $\sim 30\eta'$ mesons per event) one should expect a 100 or so of "incoherent" pions to be produced from the bubble. In the second scenario (inflation/deflation) for bubble decay the effect would be proportional to R^3 rather than R^2 and because all the pions from the bubble incoherent, therefore their number could be similar or even larger.

Naively this number represents relatively small fraction of the total number of pions in each given event. How-

[§]There are delayed pions due to "natural cocktail" of long lived resonances.

ever, the bubble is local in rapidity. Even at mid-rapidity at RHIC, with total pion multiplicity about 1000 per unit rapidity, the products of "naturally occurred" long lived resonances make about 300 of those. Additional 100 incoherent pions expected from the bubble makes it 400, with the average λ changing from $.7^2 = 0.5$ to $.6^2 = 0.36$, not a non-negligible effect.

Therefore, we propose to look at the event-by-event fluctuations of parameter λ_*, hunting for the tail of the distribution toward its values *smaller* then the average, preferably at low p_t. An unusually long tail may indicate the bubble formation. The thermodynamical fluctuations in the particle composition are expected to be very small $O(1/\sqrt{N})$, effect, obviously unable to produce any long tail by itself.

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APPENDIX A.

Here is the definition of $\delta U_{\pi^\pm}(z)$ which appears in eq.(22) in terms of the profile functions $\phi_S(z), \phi_T(z)$:

$$\delta U_{\pi^\pm}(z)\pi_i = \quad (25)$$

$$\left(\frac{\phi_T(z)}{2\sin\frac{\phi_T(z)}{2}}\right)^2 \left[\phi_T \frac{d}{dz}\left(\frac{f}{\phi_T}\right)\frac{d}{dz} - \frac{d}{dz}\left(\frac{f}{\phi_T}\frac{d\phi_T}{dz}\right)\right]\pi_i$$

with $f(z)$ defined as

$$f \equiv 1 - \left(\frac{2}{\phi_T}\sin\frac{\phi_T(z)}{2}\right)^2 \qquad (26)$$

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